Mechanical Systems Laboratory: Lecture 8
Brief Review of Stability; PD Position Control of a Robot Arm

1. Brief Review of Stability

Stability refers to the concept of whether a system's performance "blows up" or converges to some value. What are some applications in which stability analysis is very important?

Three types of stability are:

The location of the poles of the transfer function determine the type of stability.
Why? Consider a second-order system:

Remember, the inverse Laplace transform of the transfer function is the impulse response:

Case 1: $Re\{p_i\} < 0$		
Case 2: $Re\{p_i\} = 0$		
Case 3: $Re\{p_i\} > 0$		

So the location of the poles in the complex plane determines the type of response of the system.

Exercise 1: Label the complex plane with the following words:
stable, unstable, marginally stable, oscillation, no oscillation, faster, slower, higher frequency oscillation, lower frequency oscillation

2. PD Position Control of a Robot Arm (P = proportional, D = Derivative)

Position control – most common industrial control system
Can you think of some applications?

Consider a one-joint robot arm:

Assume: 1) no friction or gravity; 2) we have a controller that can apply any torque that we want; 3) we can sense θ (for example, with a potentiometer)

Exercise 2: Design a proportional feedback controller to position the robot arm at $\theta=\theta_d$, find its transfer function, and analyze its stability

Exercise 3: Design a way to fix the problem. What kind of hardware would you need?

Two approaches to sensing angular velocity:
1)
2)

What are the dynamics and transfer function of the robot with the new controller?

How are the gains K_p and K_v related to the natural frequency and damping ratio?

What is the step response of the system?

$\zeta = 0$ $\quad\theta(t) = 1 - \cos(\omega_n t)$

$0 < \zeta < 1$ $\quad\theta(t) = 1 - \dfrac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \tan^{-1}\dfrac{\sqrt{1-\zeta^2}}{\zeta}) \qquad \omega_d = \omega_n\sqrt{1-\zeta^2}$

$\zeta = 1$ $\quad\theta(t) = 1 - e^{-\omega_n t}(1+\omega_n t)$

$\zeta > 1$ sum of two exponentials – see book; can neglect one exponential if $\zeta > 2$

Note: the damping ratio determines whether the system oscillates (i.e. whether the poles have an imaginary part)

Exercise 4: Plot what the step response of the system would look like for different values of the damping ratio and natural frequency.

Notes: Overdamped systems are "sluggish"
Among systems responding without overshoot, critically damped systems exhibit the fastest response.
Underdamped systems with $0.5 < \zeta < .8$ get close to final value more rapidly than critically damped or overdamped systems.
The settling time of an underdamped (or critically damped) system is:

Exercise 5: Given a one-joint robot arm (no friction, no gravity) with $J = 1\ \text{kgm}^2$. Design a PD position controller such that the robot finishes 95% of a commanded step-function movement in .5 seconds, with no overshoot.